Low-rank Parareal: a low-rank parallel-in-time integrator

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Abstract

The Parareal algorithm of Lions, Maday, and Turinici is a well-known time parallel algorithm for evolution problems. It is based on a Newton-like iteration, with cheap coarse corrections performed sequentially, and expensive fine solves performed in parallel.
On the other hand, the dynamical low-rank approximation (DLRA), proposed by Koch and Lubich, is a recent technique that allows for solving large-scale problems on the manifold of low-rank matrices. In particular, the method has a low memory footprint and is much faster than dense techniques. The cost and accuracy are mostly governed by the rank chosen for the approximation. Due to its large-scale purpose, having a time parallel algorithm would be particularly interesting.
After introducing the DLRA, we will see how to exploit its properties and get a low-rank Parareal algorithm. The new technique has both linear and superlinear convergence bounds, which are verified numerically. If time allows, we'll introduce a Krylov-based method for solving the DLRA, which is particularly useful for parabolic problems.

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A unified analysis framework for iterative Parallel-in-Time algorithms

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Abstract

Recent advances in supercomputing architectures in the last decades have pushed the scientific community to develop more efficient algorithms for high performance computing (HPC).
In particular, the switch from improving processor speed to increasing concurrency in HPC have motivated the development of new parallelization algorithms that can harness the computing power of massively parallel HPC architectures.
For two decades, there have been research efforts in developing parallel computing capabilities for time integration methods used the simulation of time dependent problems (Numerical Weather Prediction, Computational Fluid Dynamics, Fluid-Structure interactions, ...).
However, to develop such "Parallel-in-Time" algorithms (or PinT), one needs to deal with the sequential nature of time: it is necessary to know the past and the present before computing the future.
Hence, computing both past, present and future in parallel needs the development of new algorithms with different computation paradigms as the classical ones used for time-dependent problems.

Over the last decades, many research efforts have studied iterative Parallel-in-Time (PinT) algorithms, in particular Parareal (Lions, Maday, Turinici), PFASST (Minion and Emmett), MGRIT (Falgout, Friedhoff, Kolev, MacLachlan, Schroder) and a specific form of Space-Time Multigrid (Gander and Neumueller).
While various convergence analyses exist for each algorithm separately, it is difficult to connect them and compare convergence of these iterative PinT methods when applied to various model problems, and in applications.
In this talk, I will present a new approach that lets us analyze the convergence of these four iterative algorithms in a single framework. Following an idea of Gander and Hairer already used to analyze Parareal convergence, this framework is based on an abstract view of each iterative PinT algorithm and provides understanding of their different convergence mechanisms. We use it to show some key similarities and differences between all those iterative methods while focusing on the Dahlquist equation, the fundamental time-dependent test problem.

*Speaker